Magilatin generating functions and sequences

(general form, with cubic data)

Notation:

L, S: magilatin, semimagic squares (all positive values).

ml: magilatin, except in g.f.'s.

l, s: normalized squares (symmetry types).

R: reduced squares (least element is 0).

r: reduced normalized squares (reduced symmetry types).

n: semimagic r.

gf: generating function in some form.

gfsum: generating function as a sum of simple terms.

c: Cubic (fixed strict upper bound; weak upper bound for reduced).

a: Affine (fixed magic sum).

p: Period of the quasipolynomial (known from geometry). (Period of the truncated quasipolynomial, in the affine count.)
d: Dimension of the geometry = degree of the quasipolynomials.
RtoLfactor: the rational function that multiplies Rgf to Lgf and rgf to lgf.

enddegree: The number of terms desired in the sequences, from degree 1 (but initial zeros will be omitted).

The number of terms desired of each sequence is "enddegree".

```
> enddegree:=500;
```

$$enddegree := 500$$

This is for cubic.

```
> d:=5; p:=60;
  RtoLfactor:=x^2/(1-x)^2;
```

$$d := 5$$

$$p := 60$$

$$RtoLfactor := \frac{x^2}{(1-x)^2}$$

We start by recomputing r_s=rsgf from the semimagic count. From the Latte results we get the closed Ehrhart g.f. of each flat, which depends on whether we're doing cubic or affine.

Set up the simplex data for the faces and intersection polytopes in the semimagic part of the magilatin series.

```
> simplexname[1]:="OABC": ehrgf[1]:= 1/((1-x)^3*(1-x^2)) : dimen[1]:=3:
  simplexname[2]:="OEE2": ehrgf[2]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
  dimen[2]:=2:
  simplexname[3]:="OAE2": ehrgf[3]:= 1/((1-x)*(1-x^2)^2) : dimen[3]:=2:
  simplexname[4]:="ADE2": ehrgf[4]:= 1/((1-x^2)^3) : dimen[4]:=2:
  simplexname[5]:="DE1E2": ehrgf[5]:= 1/((1-x^2)^2*(1-x^3)) : dimen[5]:=2:
  simplexname[6]:="OCE": ehrgf[6]:= 1/((1-x)^2*(1-x^3)) : dimen[6]:=2:
```

```
simplexname[7]:="BDE1": ehrgf[7]:= 1/((1-x)*(1-x^2)*(1-x^3)) :
dimen[7]:=2:
simplexname[8]:="ABD": ehrgf[8]:= 1/((1-x)*(1-x^2)^2) : dimen[8]:=2:
simplexname[9]:="FG1": ehrgf[9]:= 1/((1-x^3)*(1-x^5)) : dimen[9]:=1:
simplexname[10]:="EF": ehrgf[10]:= 1/((1-x^3)^2) : dimen[10]:=1:
simplexname[11]:="OG": ehrgf[11]:= 1/((1-x)*(1-x^4)) : dimen[11]:=1:
simplexname[12]:="FG": ehrgf[12]:= 1/((1-x^3)*(1-x^4)) : dimen[12]:=1:
simplexname[13]:="AF": ehrgf[13]:= 1/((1-x^2)*(1-x^3)) : dimen[13]:=1:
simplexname[14]:="DG": ehrgf[14]:= 1/((1-x^2)*(1-x^4)) : dimen[14]:=1:
simplexname[15]:="DG2": ehrgf[15]:= 1/((1-x^2)*(1-x^5)) : dimen[15]:=1:
simplexname[16]:="DE": ehrgf[16]:= 1/((1-x^2)*(1-x^3)) : dimen[16]:=1:
simplexname[17]:="H": ehrgf[17] := 1/(1-x^5) : dimen[17]:=0:
```

The closed E.g.f. is converted to the open E.g.f. The first step is to compute the Mobius function of the intersection poset.

```
> for n from 1 to 17 do
    mu[n]:=(-1)^(dimen[1]-dimen[n]):
  od:
  mu[14]:=2*mu[14]:
  for n from 1 to 17 do
    openehrgf[n]:=simplify(-(-1)^dimen[n]*subs(x=1/x,ehrgf[n])):
  od:
```

```
> for n from 1 to 17 do
    rsgfterm[n]:=openehrgf[n]:
  od:
  rsgfsum:=sum(mu[nn]*rsgfterm[nn],nn=1..17):
  rsgf:=simplify(rsgfsum):
  sgf:=simplify(RtoLfactor*rsgf):
```

The additional faces and intersection polytopes involved in the magilatin computation. These depend on whether we're cubic or affine.

```
> mlsimplexname[1]:="OAB": mlehrgf[1]:= 1 / ((1-x)^2*(1-x^2)) :
  mldimen[1]:=2  :
  mlsimplexname[2]:="OE": mlehrgf[2]:= 1 / ((1-x)*(1-x^3)) : mldimen[2]:=1
  :
  mlsimplexname[3]:="OAC": mlehrgf[3]:= 1 / ((1-x)^2*(1-x^2)) :
  mldimen[3]:=2  :
  mlsimplexname[4]:="AD": mlehrgf[4]:= 1/(1-x^2)^2 : mldimen[4]:=1 :
  mlsimplexname[5]:="DE1": mlehrgf[5]:= 1 / ((1-x^2)*(1-x^3)) :
  mldimen[5]:=1  :
  mlsimplexname[6]:="OBC": mlehrgf[6]:= 1/(1-x)^3 : mldimen[6]:=2 :
  mlsimplexname[7]:="OE2": mlehrgf[7]:= 1 / ((1-x)*(1-x^2)) :
  mldimen[7]:=1  :
  mlsimplexname[8]:="BD": mlehrgf[8]:= 1 / ((1-x)*(1-x^2)) : mldimen[8]:=1
  :
  mlsimplexname[9]:="DE2": mlehrgf[9]:= 1/(1-x^2)^2 : mldimen[9]:=1 :
  mlsimplexname[10]:="F": mlehrgf[10]:= 1/(1-x^3) : mldimen[10]:=0 :
  mlsimplexname[11]:="OB": mlehrgf[11]:= 1/(1-x)^2 : mldimen[11]:=1 :
```

Now a general computation. First, open Ehrhart g.f.'s.

```
> for n from 1 to 11 do
    openmlehrgf[n]:=simplify(-(-1)^mldimen[n]*subs(x=1/x,mlehrgf[n])):
  od:
```

(-1)^3 n_OAB(1/x) equals mlehrgf[1]+mlehrgf[2], and hence n_OAB(x) is, by another method that gives a nicer appearance, summing mu(,)E^o(x):

```
> mlnnew[1] := openmlehrgf[1]-openmlehrgf[2]:
```

(-1)^3 n_OAC(1/x) equals mlehrgf[3]+mlehrgf[4]+mlehrgf[5]. Hence n_OAC(x) equals

```
> mlnnew[2] := openmlehrgf[3]-openmlehrgf[4]-openmlehrgf[5]:
```

(-1)^3 n_OBC(1/x) equals mlehrgf[6]+mlehrgf[7]+mlehrgf[8]+mlehrgf[9]+mlehrgf[10]. So n_OBC(x) equals

```
> mlnnew[3] :=
  openmlehrgf[6]-openmlehrgf[7]-openmlehrgf[8]-openmlehrgf[9]+openmlehrgf[
  10]:
```

Finally, OB gives mlehrgf[11], so that n_OB(x) is

```
> mlnnew[4] := openmlehrgf[11]:
```

To compute r, we need rs=n from semimagic, which equals rgf:

```
> Rgfsum:=72*rsgfsum+36*(mlnnew[1]+mlnnew[2]+mlnnew[3])+12*mlnnew[4]:
  Rgf:=simplify(Rgfsum);
```

$$Rgf := \frac{1}{(x^4-1)(x^5-1)(x^3-1)^2(x+1)^2}(12x^2(79x^{15}+190x^{14}+260x^{13}+250x^{12}+211x^{11}+179x^{10}+181x^9+198x^8+210x^7+181x^6+125x^5+61x^4+22x^3+8x^2+4x+1))$$

Hence L, the g.f. of the number of magilatin squares, equals

```
> Lgf:=simplify(RtoLfactor*Rgf);
```

$$Lgf := \frac{1}{(x-1)^2(x^4-1)(x^5-1)(x^3-1)^2(x+1)^2}(12x^4(79x^{15}+190x^{14}+260x^{13}+250x^{12}+211x^{11}+179x^{10}+181x^9+198x^8+210x^7+181x^6+125x^5+61x^4+22x^3+8x^2+4x+1))$$

Now compute the number of reduced symmetry types:

```
> rgfsum:=rsgfsum+mlnnew[1]+mlnnew[2]+mlnnew[3]+mlnnew[4]:
  rgf:=simplify(rgfsum);
```

$$rgf := \frac{1}{(x^4-1)(x^5-1)(x^3-1)^2(x+1)^2}(x^2(9x^{15}+20x^{14}+23x^{13}+16x^{12}+10x^{11}+13x^{10}+27x^9+43x^8+54x^7+52x^6+41x^5+25x^4+14x^3+8x^2+4x+1))$$

The g.f. of the total number of symmetry types, l_ml ("lgf"):

```
> lgf:=simplify(RtoLfactor*rgf);
```

$$lgf := \frac{1}{(x-1)^2(x^4-1)(x^5-1)(x^3-1)^2(x+1)^2}(x^4(9x^{15}+20x^{14}+23x^{13}+16x^{12}+10x^{11}+13x^{10}+27x^9+43x^8+54x^7+52x^6+41x^5+25x^4+14x^3+8x^2+4x+1))$$

Generate the series expansions of the g.f.'s.

Expressing the rational function with standard denominator gives an orders-of-magnitude speedup in the series expansion.

Standard denominator (1-x^p)^{d+1}.

```
> pdenom:=(1-x^p):
  standenom:=pdenom^(d+1);
```

$$\mathit{standenom} := (1 - x^{60})^6$$

G.f. as rational function with standard denominator.

```
> Lgfstandnum:=simplify(numer(Lgf)*simplify(standenom/denom(Lgf))):
  Lgf:=Lgfstandnum/standenom;
```

$$\mathit{Lgf} := \frac{1}{(1 - x^{60})^6} (12 x^4 (79 x^{15} + 190 x^{14} + 260 x^{13} + 250 x^{12} + 211 x^{11} + 179 x^{10} + 181 x^9 + 198 x^8 + 210 x^7 + 181 x^6 + 125 x^5 + 61 x^4 + 22 x^3 + 8 x^2 + 4 x + 1) (1 - x + x^3 - x^5 + x^6 + x^{15} + x^{12} - x^{13} - x^{17} + x^{18} + x^{54} - x^{53} + x^{51} - x^{49} + x^{48} + x^{42} - x^{41} + x^{39} - x^{37} + x^{36} + x^{30} - x^{29} + x^{27} - x^{25} + x^{24}) (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{22} + x^{20} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}) (1 + x + x^2 + x^3 + x^5 + x^4 + x^6 + x^7 + x^8 + x^{10} + x^{15} + x^{12} + x^9 + x^{13} + x^{11} + x^{19} + x^{14} + x^{16} + x^{17} + x^{18} + x^{22} + x^{20} + x^{21} + x^{59} + x^{57} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23})^{\wedge}2 (1 - x + x^2 - x^3 + x^4 + x^{10} + x^{12} - x^{13} - x^{11} + x^{14} + x^{22} + x^{20} - x^{21} + x^{54} - x^{53} + x^{52} - x^{51} + x^{50} + x^{44} - x^{43} + x^{42} - x^{41} + x^{40} + x^{34} - x^{33} + x^{32} - x^{31} + x^{30} + x^{24} - x^{23}))$$

G.f. as rational function with standard denominator.

```
> Rgfstandnum:=simplify(numer(Rgf)*standenom/denom(Rgf)):
  Rgf:=Rgfstandnum/standenom;
```

$$\mathit{Rgf} := \frac{1}{(1 - x^{60})^6} (12 (79 x^{15} + 190 x^{14} + 260 x^{13} + 250 x^{12} + 211 x^{11} + 179 x^{10} + 181 x^9 + 198 x^8 + 210 x^7 + 181 x^6 + 125 x^5 + 61 x^4 + 22 x^3 + 8 x^2 + 4 x + 1) x^2 (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^9 + x^6 + x^3 + 1)^{\wedge}2 (-1 + x - x^2 + x^3 + x^5 - x^4 - x^6 + x^7 - x^8 - x^{10} + x^{15} - x^{12} + x^9 + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{18} - x^{22} - x^{20} + x^{21} + x^{59} + x^{57} - x^{58} - x^{56} + x^{55} - x^{54} + x^{53} - x^{52} + x^{51} - x^{50} + x^{49} - x^{48} + x^{47} - x^{46} + x^{45} - x^{44} + x^{43} - x^{42} + x^{41} - x^{40} + x^{39} - x^{38} + x^{37} - x^{36} + x^{35} - x^{34} + x^{33} - x^{32} + x^{31} - x^{30} + x^{29} - x^{28} + x^{27} - x^{26} + x^{25} - x^{24} + x^{23}) (-1 + x^{60}) (1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{22} + x^{20} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^8 - x^6 + x^4 - x + 1))$$

G.f. as rational function with standard denominator.

```
> lgfstandnum:=simplify(numer(lgf)*simplify(standenom/denom(lgf))):
  lgf:=lgfstandnum/standenom;
```

$$\mathit{lgf} := \frac{1}{(1 - x^{60})^6} (x^4 (9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^9 + 43 x^8 + 54 x^7 + 52 x^6 + 41 x^5 + 25 x^4 + 14 x^3 + 8 x^2 + 4 x + 1) (1 - x + x^3 - x^5 + x^6 + x^{15} + x^{12} - x^{13} - x^{17} + x^{18} + x^{54}$$

$$- x^{53} + x^{51} - x^{49} + x^{48} + x^{42} - x^{41} + x^{39} - x^{37} + x^{36} + x^{30} - x^{29} + x^{27} - x^{25} + x^{24}) (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^{9} + x^{6} + x^{3} + 1)$$

$$(1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{22} + x^{20} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}) (1 + x + x^{2} + x^{3} + x^{5} + x^{4} + x^{6} + x^{7} + x^{8} + x^{10} + x^{15} + x^{12} + x^{9} + x^{13} + x^{11} + x^{19} + x^{14} + x^{16} + x^{17} + x^{18} + x^{22} + x^{20} + x^{21} + x^{59} + x^{57} + x^{58} + x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{49} + x^{48} + x^{47} + x^{46} + x^{45} + x^{44} + x^{43} + x^{42} + x^{41} + x^{40} + x^{39} + x^{38} + x^{37} + x^{36} + x^{35} + x^{34} + x^{33} + x^{32} + x^{31} + x^{30} + x^{29} + x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23})\verb|^|2 (1 - x + x^{2} - x^{3} + x^{4} + x^{10} + x^{12} - x^{13} - x^{11} + x^{14} + x^{22} + x^{20} - x^{21} + x^{54} - x^{53} + x^{52} - x^{51} + x^{50} + x^{44} - x^{43} + x^{42} - x^{41} + x^{40} + x^{34} - x^{33} + x^{32} - x^{31} + x^{30} + x^{24} - x^{23}))$$

G.f. as rational function with standard denominator.

```
> rgfstandnum:=simplify(numer(rgf)*standenom/denom(rgf)):
  rgf:=rgfstandnum/standenom;
```

$$rgf := \frac{1}{(1 - x^{60})^{6}} ((9 x^{15} + 20 x^{14} + 23 x^{13} + 16 x^{12} + 10 x^{11} + 13 x^{10} + 27 x^{9} + 43 x^{8} + 54 x^{7} + 52 x^{6} + 41 x^{5} + 25 x^{4} + 14 x^{3} + 8 x^{2} + 4 x + 1) x^{2} (x^{57} + x^{54} + x^{51} + x^{48} + x^{45} + x^{42} + x^{39} + x^{36} + x^{33} + x^{30} + x^{27} + x^{24} + x^{21} + x^{18} + x^{15} + x^{12} + x^{9} + x^{6} + x^{3} + 1)\verb|^|2 (-1 + x - x^{2} + x^{3} + x^{5} - x^{4} - x^{6} + x^{7} - x^{8} - x^{10} + x^{15} - x^{12} + x^{9} + x^{13} + x^{11} + x^{19} - x^{14} - x^{16} + x^{17} - x^{18} - x^{22} - x^{20} + x^{21} + x^{59} + x^{57} - x^{58} - x^{56} + x^{55} - x^{54} + x^{53} - x^{52} + x^{51} - x^{50} + x^{49} - x^{48} + x^{47} - x^{46} + x^{45} - x^{44} + x^{43} - x^{42} + x^{41} - x^{40} + x^{39} - x^{38} + x^{37} - x^{36} + x^{35} - x^{34} + x^{33} - x^{32} + x^{31} - x^{30} + x^{29} - x^{28} + x^{27} - x^{26} + x^{25} - x^{24} + x^{23}) (-1 + x^{60}) (1 + x^{2} + x^{4} + x^{6} + x^{8} + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{22} + x^{20} + x^{58} + x^{56} + x^{54} + x^{52} + x^{50} + x^{48} + x^{46} + x^{44} + x^{42} + x^{40} + x^{38} + x^{36} + x^{34} + x^{32} + x^{30} + x^{28} + x^{26} + x^{24}) (x^{52} - x^{51} + x^{48} - x^{46} + x^{44} - x^{41} + x^{40} + x^{32} - x^{31} + x^{28} - x^{26} + x^{24} - x^{21} + x^{20} + x^{12} - x^{11} + x^{8} - x^{6} + x^{4} - x + 1))$$

Expand the series to find the first few values of the number of squares.

```
> Lseries:=series(Lgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand the series to find the first few values of the number of reduced squares.

```
> Rseries:=series(Rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand the series to find the first few values of the number of symmetry types.

```
> lseries:=series(lgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Expand the series to find the first few values of the number of reduced symmetry types.

```
> rseries:=series(rgf,x=0,enddegree+1):
  print("Series computed.");
```

"Series computed."

Find the counting sequences

Generate the labelled sequence of magilatin square numbers of all four kinds. The first step is to compute the degree of the first non-zero term.

```
> Lgfdegree:=ldegree(numer(Lgf),x);
  Rgfdegree:=ldegree(numer(Rgf),x);
  lgfdegree:=ldegree(numer(lgf),x);
  rgfdegree:=ldegree(numer(rgf),x);
```

$$Lgfdegree := 4$$

$$Rgfdegree := 2$$

$$lgfdegree := 4$$

$$rgfdegree := 2$$

List the coefficients of each series, i.e., the terms of the counting sequences.
The comment symbol # is for controlling the output. With large "enddegree" the output is huge so it's more convenient to run each sequence's output separately and copy it from the worksheet.

```
> for n from Lgfdegree to enddegree do
    co:=coeff(Lseries,x,n):
    printf("%d  %d \n",n,co);
  od:
  print("Sequence complete.",n,co);
```

```
4  12
5  48
6  120
7  384
8  1068
9  2472
10  4896
11  9072
12  15516
13  25608
14  40296
15  61608
16  91068
17  131640
18  185136
19  255960
20  346860
21  463248
22  608088
23  789240
24  1010316
25  1280544
26  1604832
27  1994064
28  2454012
29  2998656
30  3633912
31  4376064
32  5232972
33  6223080
34  7354896
```

35	8650896
36	10120236
37	11787768
38	13665096
39	15780024
40	18144876
41	20792280
42	23734848
43	27008664
44	30629580
45	34636560
46	39046104
47	43903080
48	49224924
49	55060176
50	61429440
51	68385072
52	75948540
53	84179040
54	93098760
55	102771072
56	113222268
57	124519608
58	136690320
59	149809728
60	163905852
61	179058552
62	195300024
63	212715096
64	231337260
65	251259768
66	272516832
67	295206888
68	319369404
69	345108000
70	372462936
71	401547624
72	432403260
73	465148944
74	499831344
75	536575248
76	575428620
77	616526544
78	659917992
79	705744528
80	754061100
81	805015320
82	858663072
83	915163776
84	974574684
85	1037061624
86	1102688184
87	1171626888
88	1243942908

89	1319821224
90	1399327872
91	1482655176
92	1569876156
93	1661190336
94	1756672104
95	1856534376
96	1960852908
97	2069848032
98	2183602992
99	2302346112
100	2426162076
101	2555293392
102	2689825752
103	2830010160
104	2975940588
105	3127875960
106	3285911616
107	3450321744
108	3621203340
109	3798839304
110	3983334840
111	4174981704
112	4373886684
113	4580357736
114	4794503088
115	5016639912
116	5246885580
117	5485566192
118	5732800776
119	5988932856
120	6254082972
121	6528604224
122	6812626368
123	7106512512
124	7410394428
125	7724653008
126	8049421464
127	8385090912
128	8731804860
129	9089964648
130	9459715296
131	9841477296
132	10235397324
133	10641906600
134	11061162312
135	11493606408
136	11939398092
137	12398998968
138	12872569968
139	13360584216
140	13863213660
141	14380942512

142	14913944376
143	15462724632
144	16027458972
145	16608664224
146	17206527456
147	17821577232
148	18454002924
149	19104354912
150	19772824152
151	20459973408
152	21166005660
153	21891495912
154	22636649232
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157	24994905816
158	25823164008
159	26673343320
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161	28440777624
162	29358902976
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164	31266411516
165	32256704880
166	33271821048
167	34312457928
168	35378856876
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173	41116378752
174	42348327336
175	43609762272
176	44900956812
177	46222690296
178	47575238352
179	48959407104
180	50375474412
181	51824261016
182	53306059032
183	54821704248
184	56371491516
185	57956283768
186	59576378016
187	61232652456
188	62925419436
189	64655572416
190	66423425976
191	68229902088
192	70075317708
193	71960610576
194	73886113200
195	75852779088

196	77860943484
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198	82005053064
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200	86323788540
201	88550428824
202	90822612000
203	93141383808
204	95507103180
205	97920832344
206	100382946648
207	102894525096
208	105455946060
209	108068319720
210	110732026752
211	113448194760
212	116217221484
213	119040251808
214	121917686280
215	124850701896
216	127839702012
217	130885881120
218	133989660144
219	137152251648
220	140374079436
221	143656388976
222	146999606520
223	150404996112
224	153873002364
225	157404907320
226	161001158400
227	164663071632
228	168391097532
229	172186570920
230	176049960600
231	179982620328
232	183985021932
233	188058554088
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235	196421842152
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239	214041032472
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242	228067118016
243	232902602304
244	237819508524
245	242819376048
246	247902730488
247	253071131520
248	258325125132
249	263666291304

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252	280221524412
253	285921172872
254	291712924008
255	297598437288
256	303578288796
257	309654176376
258	315826679280
259	322097516952
260	328467289740
261	334937738256
262	341509465944
263	348184253304
264	354962707308
265	361846629984
266	368836649760
267	375934590480
268	383141084316
269	390457995648
270	397885959672
271	405426863232
272	413081363628
273	420851370024
274	428737543248
275	436741833936
276	444864906444
277	453108733944
278	461474003400
279	469962711096
280	478575547596
281	487314551448
282	496180416384
283	505175204568
284	514299633132
285	523555787280
286	532944387672
287	542467562856
288	552126037308
289	561921963408
290	571856088960
291	581930590320
292	592146219036
293	602505195744
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295	623657703552
296	634454259036
297	645400211064
298	656496342864
299	667744968960
300	679146876252
301	690704403960
302	702418363320
303	714291118680

304	726323485452
305	738517873848
306	750875102880
307	763397608104
308	776086233948
309	788943441312
310	801970078296
311	815168653032
312	828540017436
313	842086705488
314	855809594736
315	869711245008
316	883792538028
317	898056081360
318	912502760616
319	927135210000
320	941954341260
321	956962814808
322	972161546208
323	987553245120
324	1003138831356
325	1018921041144
326	1034900820792
327	1051080933384
328	1067462329692
329	1084047822696
330	1100838366912
331	1117836802824
332	1135044112092
333	1152463162560
334	1170094940136
335	1187942363496
336	1206006422796
337	1224290064288
338	1242794305776
339	1261522121664
340	1280474534076
341	1299654569040
342	1319063252568
343	1338703639344
344	1358576783820
345	1378685768760
346	1399031652864
347	1419617571600
348	1440444588204
349	1461515867016
350	1482832499640
351	1504397679432
352	1526212502460
353	1548280215720
354	1570601919600
355	1593180890472
356	1616018258028

357	1639117327728
358	1662479233800
359	1686107336568
360	1710002774652
361	1734168938112
362	1758606994944
363	1783320365376
364	1808310222300
365	1833580041168
366	1859130999192
367	1884966602208
368	1911088057884
369	1937498902440
370	1964200347936
371	1991195987184
372	2018487036780
373	2046077120424
374	2073967485384
375	2102161786248
376	2130661275180
377	2159469663864
378	2188588209072
379	2218020654168
380	2247768287100
381	2277834882480
382	2308221732792
383	2338932671256
384	2369968995324
385	2401334569824
386	2433030723744
387	2465061353808
388	2497427794188
389	2530134000864
390	2563181312472
391	2596573717536
392	2630312586876
393	2664401941416
394	2698843156944
395	2733640314576
396	2768794795068
397	2804310712152
398	2840189479272
399	2876435243352
400	2913049422876
401	2950036225752
402	2987397075072
403	3025136212440
404	3063255094428
405	3101757995760
406	3140646377976
407	3179924577864
408	3219594062220
409	3259659201744
410	3300121496640

411	3340985351664
412	3382252272204
413	3423926726016
414	3466010223528
415	3508507266912
416	3551419400940
417	3594751161912
418	3638504099856
419	3682682815296
420	3727288863372
421	3772326879384
422	3817798452888
423	3863708254392
424	3910057879068
425	3956852062008
426	4004092403424
427	4051783673832
428	4099927508940
429	4148528714688
430	4197588931896
431	4247113032456
432	4297102662444
433	4347562729680
434	4398494915952
435	4449904165008
436	4501792164252
437	4554163923888
438	4607021136648
439	4660368849456
440	4714208791260
441	4768546045272
442	4823382345696
443	4878722843712
444	4934569279212
445	4990926840024
446	5047797302616
447	5105185891752
448	5163094389804
449	5221528090152
450	5280488780352
451	5339981791368
452	5400008947980
453	5460575618592
454	5521683633672
455	5583338431176
456	5645541847260
457	5708299357536
458	5771612835888
459	5835487796160
460	5899926117996
461	5964933385584
462	6030511483896
463	6096666035856
464	6163398964956

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465  6230715932280
466  6298618867008
467  6367113501648
468  6436201771356
469  6505889447592
470  6576178503960
471  6647074751016
472  6718580168268
473  6790700638632
474  6863438147376
475  6936798616872
476  7010784071772
477  7085400473616
478  7160649853032
479  7236538245144
480  7313067686412
481  7390244251776
482  7468070017152
483  7546551097728
484  7625689575756
485  7705491640368
486  7785959379576
487  7867099022976
488  7948912699116
489  8031406678056
490  8114583094176
491  8198448292848
492  8283004414428
493  8368257845256
494  8454210766440
495  8540869605288
496  8628236549244
497  8716318101432
498  8805116455344
499  8894638155864
500  8984885437740
```

"Sequence complete.", 501, 8984885437740

```
> for n from Rgfdegree to enddegree do
   co:=coeff(Rseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Sequence complete.",n,co);
```

```
2  12
3  24
4  36
5  192
6  420
7  720
8  1020
9  1752
10  2268
11  3648
```

12	4596
13	6624
14	8148
15	11112
16	12924
17	17328
18	20076
19	25488
20	28452
21	36312
22	39924
23	49152
24	54060
25	64944
26	70716
27	84696
28	90612
29	106896
30	114756
31	133200
32	141708
33	164184
34	173340
35	198192
36	209796
37	237600
38	249924
39	282552
40	295164
41	331248
42	347100
43	386064
44	402564
45	447432
46	464868
47	513408
48	534012
49	586368
50	607836
51	667032
52	689220
53	752592
54	778884
55	846144
56	873372
57	948696
58	976716
59	1056576
60	1088772
61	1173600
62	1207092
63	1300344
64	1334556
65	1432992

66	1472460
67	1576080
68	1616340
69	1729752
70	1770948
71	1890048
72	1936716
73	2061504
74	2109468
75	2244552
76	2293524
77	2435088
78	2490036
79	2637648
80	2693532
81	2852952
82	2910204
83	3076032
84	3139620
85	3312144
86	3377316
87	3562296
88	3628332
89	3820656
90	3893676
91	4093200
92	4167588
93	4380504
94	4456260
95	4676592
96	4759836
97	4988160
98	5072844
99	5315352
100	5401044
101	5652048
102	5746020
103	6004944
104	6100284
105	6374472
106	6471468
107	6754368
108	6859572
109	7151328
110	7258116
111	7566072
112	7674300
113	7991472
114	8108844
115	8434944
116	8553972
117	8897496
118	9018036
119	9371136

120	9500892
121	9864000
122	9995772
123	10376664
124	10509876
125	10900992
126	11044500
127	11445840
128	11590860
129	12011352
130	12158028
131	12589248
132	12746436
133	13188384
134	13347588
135	13809192
136	13970124
137	14443248
138	14615196
139	15099408
140	15273012
141	15778392
142	15954084
143	16470912
144	16657980
145	17186544
146	17375916
147	17926296
148	18117252
149	18680016
150	18882996
151	19458000
152	19663068
153	20260824
154	20467980
155	21078192
156	21297876
157	21921120
158	22142964
159	22789752
160	23013324
161	23673648
162	23910540
163	24583824
164	24822804
165	25520712
166	25762068
167	26473728
168	26728332
169	27453888
170	27710796
171	28461912
172	28720980
173	29486352

174	29759604
175	30538944
176	30814572
177	31620696
178	31898556
179	32719296
180	33011412
181	33847200
182	34142052
183	35004984
184	35301996
185	36180192
186	36492540
187	37386000
188	37700580
189	38622552
190	38939508
191	39877248
192	40209756
193	41163264
194	41498508
195	42481032
196	42818724
197	43817808
198	44171556
199	45186768
200	45542892
201	46588632
202	46947564
203	48009792
204	48385140
205	49464144
206	49842516
207	50952696
208	51333372
209	52460976
210	52858716
211	54003600
212	54404148
213	55581144
214	55984500
215	57178992
216	57599916
217	58812480
218	59236284
219	60481752
220	60908004
221	62172048
222	62616660
223	63898704
224	64346124
225	65662152
226	66112668
227	67447488

228	67916292
229	69270048
230	69741876
231	71130552
232	71605260
233	73013232
234	73507164
235	74934144
236	75431172
237	76894296
238	77394276
239	78877056
240	79396332
241	80899200
242	81421932
243	82961304
244	83486916
245	85046592
246	85592580
247	87172560
248	87721500
249	89339352
250	89891388
251	91530048
252	92102676
253	93762144
254	94338228
255	96036072
256	96615324
257	98334768
258	98935116
259	100675728
260	101279172
261	103059672
262	103666644
263	105468672
264	106097100
265	107920944
266	108553116
267	110417496
268	111052692
269	112939536
270	113596836
271	115506000
272	116166828
273	118117464
274	118781820
275	120754992
276	121441956
277	123438240
278	124128804
279	126167352
280	126861084

281	128923248
282	129640380
283	131725584
284	132446244
285	134574792
286	135299268
287	137451648
288	138199452
289	140375808
290	141127356
291	143347992
292	144103140
293	146348112
294	147127524
295	149396544
296	150179772
297	152494296
298	153281196
299	155620416
300	156431652
301	158796000
302	159611412
303	162021624
304	162840636
305	165276192
306	166120620
307	168581520
308	169429620
309	171937752
310	172789668
311	175323648
312	176201196
313	178761024
314	179642748
315	182250312
316	183135924
317	185770128
318	186681876
319	189342288
320	190257852
321	192967512
322	193887324
323	196623552
324	197569860
325	200332944
326	201283716
327	204096696
328	205051212
329	207891696
330	208873356
331	211741200
332	212727108
333	215645784
334	216635940

335	219582192
336	220599996
337	223574400
338	224596524
339	227622552
340	228648564
341	231703248
342	232757700
343	235840464
344	236899164
345	240034632
346	241097868
347	244262208
348	245353812
349	248547168
350	249643236
351	252890232
352	253990620
353	257266992
354	258396684
355	261702144
356	262836372
357	266196696
358	267335316
359	270725376
360	271893372
361	275313600
362	276486492
363	279961944
364	281139156
365	284644992
366	285852660
367	289388880
368	290600940
369	294193752
370	295410348
371	299034048
372	300281316
373	303935904
374	305188068
375	308899752
376	310156524
377	313899888
378	315187836
379	318962448
380	320254932
381	324088152
382	325385604
383	329250432
384	330579420
385	334476144
386	335810316
387	339766296
388	341104932

389	345093456
390	346464276
391	350485200
392	351860988
393	355942104
394	357322860
395	361436592
396	362850036
397	366996960
398	368415444
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401	378288048
402	379744620
403	384019344
404	385480884
405	389817672
406	391284468
407	395655168
408	397155372
409	401560128
410	403065516
411	407533272
412	409043700
413	413545872
414	415090644
415	419626944
416	421176972
417	425777496
418	427332636
419	431967936
420	433557492
421	438228000
422	439823172
423	444558264
424	446158476
425	450928992
426	452564700
427	457370640
428	459011460
429	463883352
430	465529428
431	470437248
432	472119036
433	477062784
434	478750188
435	483760392
436	485453124
437	490500048
438	492228996
439	497312208
440	499046412
441	504197592

442	505937484
443	511125312
444	512901780
445	518126544
446	519908916
447	525202296
448	526989852
449	532320816
450	534145596
451	539514000
452	541344468
453	546782424
454	548618580
455	554094192
456	555968076
457	561481920
458	563361564
459	568945752
460	570830724
461	576453648
462	578377140
463	584038224
464	585967404
465	591699912
466	593635068
467	599406528
468	601380132
469	607190688
470	609170196
471	615053112
472	617038380
473	622960752
474	624985404
475	630946944
476	632977572
477	639012696
478	641049156
479	647124096
480	649200012
481	655315200
482	657397452
483	663586584
484	665674596
485	671904192
486	674032740
487	680302800
488	682437180
489	688782552
490	690922908
491	697309248
492	699490356
493	705917664
494	708105108
495	714608232

```
496  716801724
497  723346608
498  725581356
499  732167568
500  734408292
```

"Sequence complete.", 501, 734408292

```
> for n from lgfdegree to enddegree do
   co:=coeff(lseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Sequence complete.",n,co);
```

```
4  1
5  4
6  10
7  24
8  53
9  106
10  191
11  328
12  528
13  822
14  1230
15  1794
16  2542
17  3534
18  4802
19  6428
20  8460
21  10996
22  14087
23  17870
24  22405
25  27850
26  34286
27  41896
28  50773
29  61148
30  73116
31  86942
32  102751
33  120840
34  141343
35  164618
36  190808
37  220306
38  253292
39  290202
40  331226
41  376872
42  427334
43  483170
44  544622
```

45	612290
46	686425
47	767714
48	856421
49	953286
50	1058620
51	1173218
52	1297403
53	1432070
54	1577552
55	1734804
56	1904219
57	2086808
58	2282977
59	2493854
60	2719856
61	2962176
62	3221292
63	3498468
64	3794200
65	4109874
66	4445996
67	4804026
68	5184546
69	5589090
70	6018251
71	6473704
72	6956055
73	7467060
74	8007404
75	8578924
76	9182323
77	9819586
78	10491430
79	11199932
80	11945895
81	12731482
82	13557509
83	14426308
84	15338714
85	16297150
86	17302542
87	18357408
88	19462696
89	20621102
90	21833586
91	23102948
92	24430248
93	25818388
94	27268447
95	28783518
96	30364699
97	32015188
98	33736190

99	35531016
100	37400891
101	39349328
102	41377566
103	43489238
104	45685701
105	47970700
106	50345611
107	52814396
108	55378454
109	58041870
110	60806160
111	63675534
112	66651530
113	69738588
114	72938266
115	76255134
116	79690880
117	83250200
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"Sequence complete.", 501, 127334079820

```
> for n from rgfdegree to enddegree do
   co:=coeff(rseries,x,n):
   printf("%d  %d \n",n,co);
  od:
  print("Sequence complete.",n,co);
```

```
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3  2
4  3
5  8
6  15
7  24
8  32
9  52
10  63
11  94
12  114
13  156
14  184
15  244
16  276
17  358
18  406
19  504
20  555
21  692
22  752
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26	1267
27	1498
28	1593
29	1858
30	1983
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32	2414
33	2772
34	2915
35	3308
36	3488
37	3924
38	4114
39	4622
40	4816
41	5374
42	5616
43	6216
44	6467
45	7154
46	7418
47	8158
48	8469
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53	11770
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55	13174
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61	18060
62	18556
63	19942
64	20448
65	21908
66	22490
67	24024
68	24617
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70	26898
71	28654
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74	31879
75	33864
76	34581

77	36658
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79	39624
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89	56878
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91	60840
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102	84791
103	88536
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179	470158
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"Sequence complete.", 501, 10324035

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